A simple rule for axon outgrowth and synaptic competition generates realistic connection lengths and filling fractions

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Supplementary Material

1. Connection length distributions

Distribution of connections lengths for different parameter sets for two-dimensional growth of 1000 or 400 neurons. The subscripted label *abc* (e.g., $w2d1000_{abc}$) indicates the growth model with 100 nodes, *a*=1 for static growth, *b*=1 for the number of connections per neuron being limited to one, and *c*=1 if the cell size is variable between 0 and 2 spatial units (compared to the uniform cell size of one in the standard simulations).

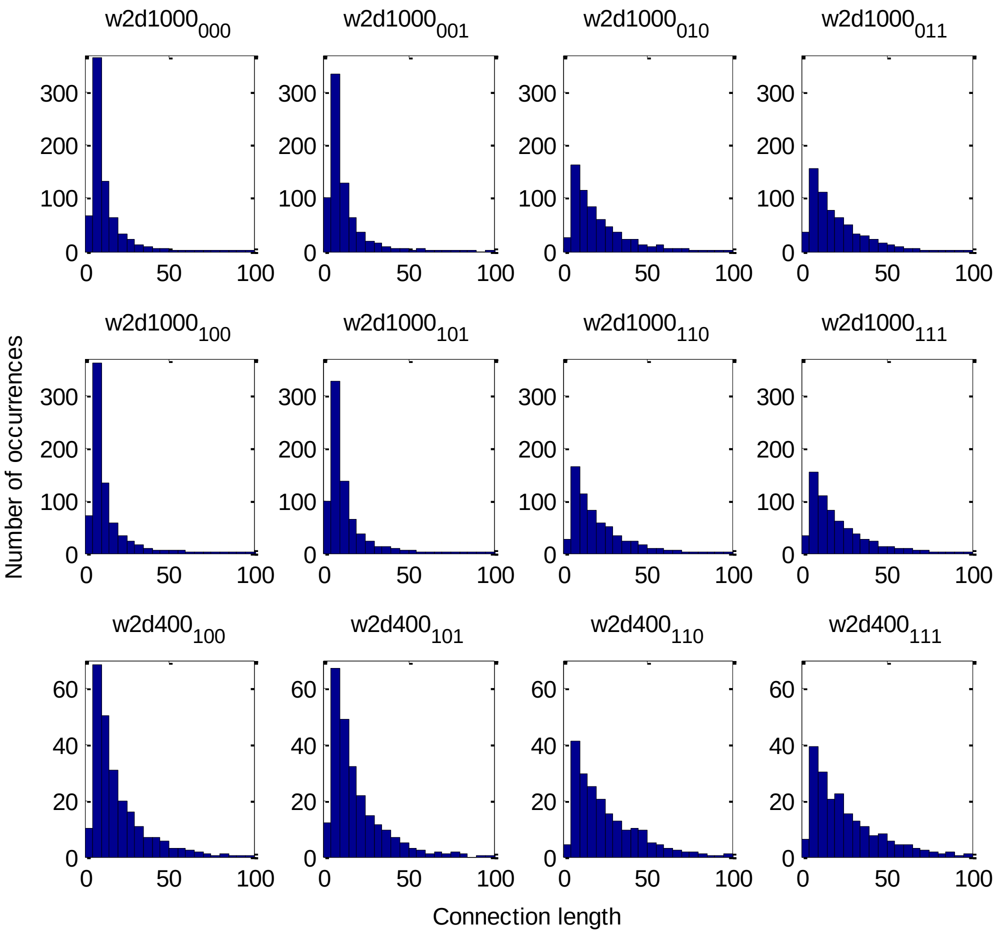

Figure S1. Connection length distributions (average over 10 generated networks).

2. Filling fraction and connection lengths for three-dimensional axon growth

We also simulated axonal growth in three dimensions. The embedding space was 34x34x34 units large so that neurons could easily be placed without an overlap of their three-dimensional shape. Figure S2 shows the connections length distributions for 400 and 1000 generated neurons following the same parameters as in Figure S1.

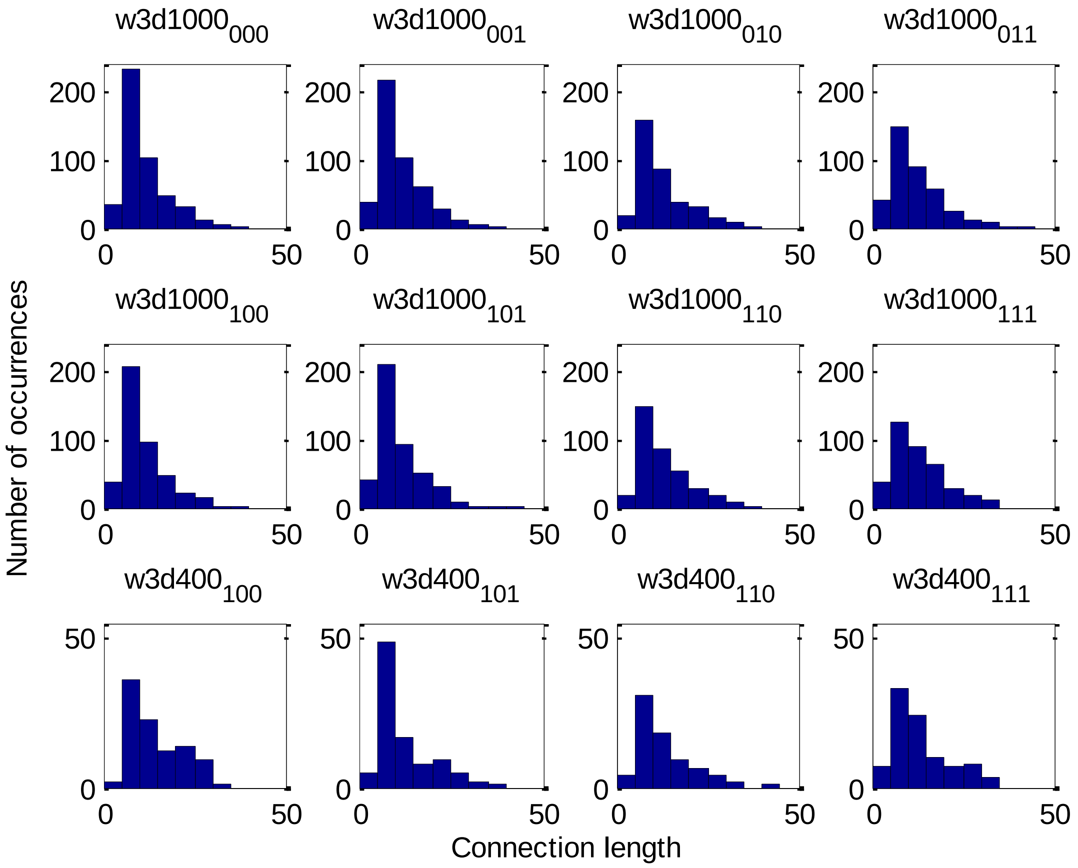

Figure S2. Connection length distribution for three-dimensional growth. Average over 10 generated networks; for the description of labels, see Figure S1.

We again tested the relation between the number of neurons (neural density) and the filling fraction and edge density of the resulting three-dimensional networks as we did for figure 6 of the main text (Figure S3). The filling fraction decreases with the neural density (Fig. S3A) as it did for the two-dimensional embedding. Note that the higher absolute filling fractions, compared to two-dimensional growth, are due to lower spatial density (for 1000 neurons, for example, 2.5% of the three-dimensional space but 10% of the two-dimensional embedding space are occupied by neurons).

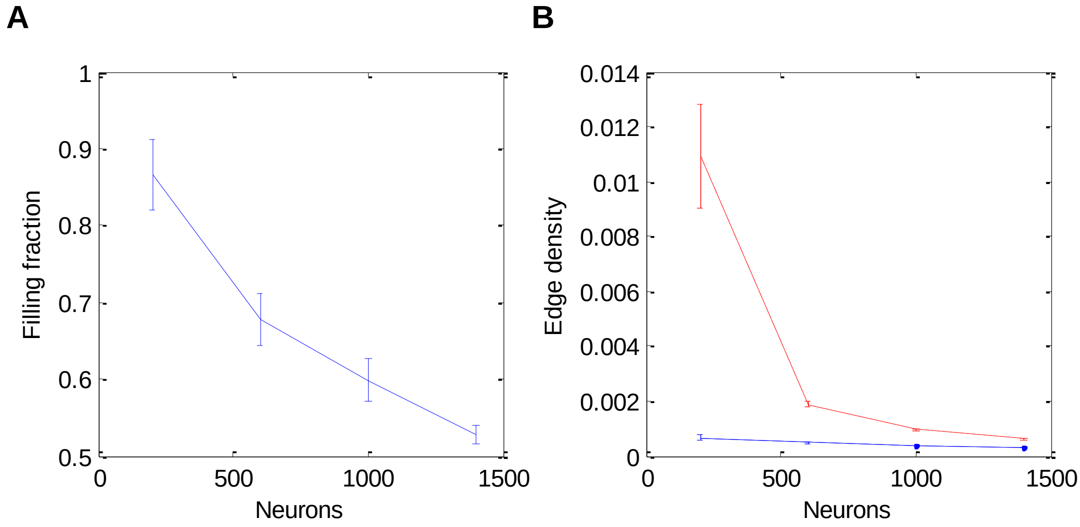

Figure S3. Dependence of filling fraction and edge density on the number of neurons for three-dimensional growth. (*A*) Filling fraction, which is the proportion of successfully established connections after axon-cell contact, depends on the number of neurons (vertical bars indicate the standard deviation while lines show the average over 10 generated networks). (*B*) The edge density–the number of existing edges divided by the number of possible edges–decreases for larger numbers of neurons (average and standard deviation as in (*A*); solid line: complete network; dashed line: network without isolated nodes).

3. Explaining the exponential tail in axon length distributions

As shown in the main text, all anatomical networks as well as the simulated networks show an exponential tail in the axon length distribution (Figure S4 A). What is the underlying reason for such a length distribution in the simulated networks?

Let us have a look at the simple case of no competition for dendritic target space. The probability to hit a neuron at each time step is given by the fraction p of two-dimensional or three-dimensional space that is occupied by neurons: for example, if 10% of the embedding space is occupied by neurons, the probability to enter a spatial area and hit a neuron is p=0.1. As for each growth step the probability p of hitting a neuron and the probability q=1-p of not hitting a neuron is the same (independent events), the probability of finding another neuron after n growth steps then follows an exponential distribution (see Figure S4 B).

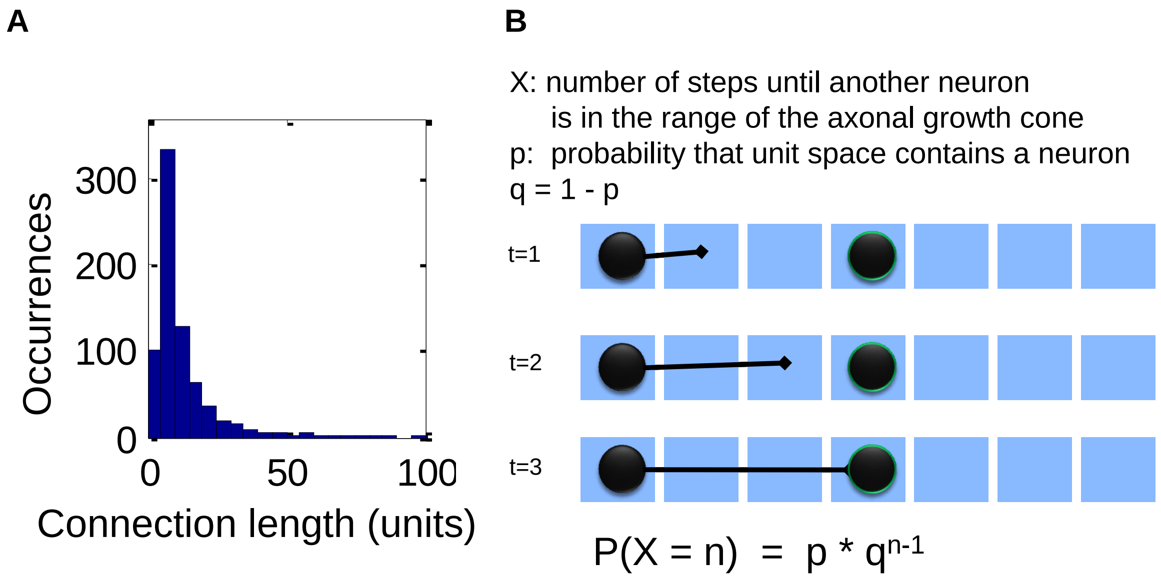

Figure S4. Mechanism for axon length distribution. (*A*) Connection length distribution from a simulation without competition for space at the target neuron. The distribution shows an exponential tail beyond short connection lengths. (*B*) Schematic overview of three time steps of axon growth. Blue squares represent spatial units and black spheres show spatial units that contain a neuron. The axonal growth cone moves along the horizontal direction testing whether each new spatial unit contains a neuron. At the third step, a neuron has been found and an axon was established (no competition). The probability $P(X=n)$ to find another neuron after n time steps follows an exponential distribution, that means, $P(X=n) = p\, q^{n-1}$.

In our simulations, the axon length distributions showed an exponential tail but also a peak in number of occurrences at lengths above zero. The latter is due to the growth steps and the way distance between neurons is calculated.